GEOMETRICAL REDUCTION AND CALCULATION OF NYLON OFFSET MEASUREMENTS

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Abstract

The estimation of an expected nylon offset measurement and the geometrical reduction of a nylon offset measurement are necessary for using and processing these types of measurements. Several years ago, failure cases for the reduction algorithm deployed for these calculations were encountered. A new algorithm was established, but now new failure cases for both the reduction and the estimation of these measurements have now been encountered.

This document presents a further development of these algorithms, with an emphasis on further limiting any failure cases, and making sure that any failure cases are identified such that they can be avoided.

Mots-clés: Écartomètrie, algorithme, réduction, estimation

Key words: Offset measurements, algorithm, reduction, estimation
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1. INTRODUCTION

The estimation of an expected nylon offset measurement and the geometrical reduction of a nylon offset measurement are necessary for using and processing these types of measurements. Several years ago, failure cases for the reduction algorithm deployed for these calculations were encountered. A new algorithm was established, but now new failure cases for both the reduction and the estimation of these measurements have now been encountered.

This document presents a further development of these algorithms, with an emphasis on further limiting any failure cases, and making sure that any failure cases are identified such that they can be avoided.

2. OFFSET MEASUREMENT DATA PROCESSING

The offset measurements are essentially measurements in a two dimensional plane. The offsets are measured from a point to a line and give the shortest distance between the two. In the field the line is materialised by a stretched nylon wire, and the distance measured is from a point to the vertical plane defined by the nylon wire and the local vertical. The position and orientation of the wire is determined by measuring its offset from two “anchor” points.

The 2D relationships can be treated by projection onto the local horizon.

2.1 General Definitions

Let us now define some parameters and derive some other necessary variables.

Referring to Fig. 1, let the local coordinate system be a 3D Cartesian coordinate systems OXYZ, where the horizontal plane is OXY. A nylon wire is presented by the line AB, and the two points P₁ and P₂ represent the anchor points. P₃ is the point from where the nylon wire is measured.

In the reference frame OXY let,

\[ P_1 = (X_1, Y_1) \]
\[ P_2 = (X_2, Y_2) \]
\[ P_3 = (X_3, Y_3) \]

Let the measurement from the anchor point P₁ to the nylon wire be \( e_1 \), and the measurement from the anchor point P₂ to the nylon wire be \( e_2 \). The horizontal distance from the point P₃ to the nylon wire is represented by a fixed offset, \( d\text{Cal}_3 \), plus the measured offset value, \( e_3 \).

We now define,

\[ dX_{12} = X_2 - X_1 \]
\[ dY_{12} = Y_2 - Y_1 \]
and,

\[ dX_{13} = X_3 - X_1 \]
\[ dY_{13} = Y_3 - Y_1 \]

Figure 1. Geometry of the Nylon Wire, Anchor and Measured Points

We can then define the horizontal distance from \( P_1 \) to \( P_2 \) to be,

\[ d_{12} = \sqrt{dX_{12}^2 + dY_{12}^2} \]

and the unit line vector from \( P_1 \) to \( P_2 \) to be,

\[ L_{11} = \begin{pmatrix} dX_{12} / d_{12} \\ dY_{12} / d_{12} \end{pmatrix} \]
Let us now define a second reference frame \( \text{oxy} \) where the x-axis passes through the point \( P_1 \), and is parallel to the stretched wire and the y-axis also passes through \( P_1 \), and forms an orthogonal system with the x-axis. In this system \( \text{oxy} \), let,

\[
P_1 = (x_1, y_1) = (0, 0)
\]

And

\[
P_2 = (x_2, y_2)
\]

where it can be seen from Figure 1 that,

\[
y_2 = e_2 - e_1
\]

\[
x_2 = d_{12} \cos(\theta)
\]

where \( e_1 \) and \( e_2 \) are the measured offsets as defined above, and \( \theta \), is defined to be the angle between the line \( P_1P_2 \) and the stretched nylon wire.

We also see from Figure 1 that,

\[
\sin(\theta) = \frac{y_2}{d_{12}}
\]

\[
\theta = \sin^{-1} \left( \frac{y_2}{d_{12}} \right)
\]

and,

\[
\cos(\theta) = \sqrt{1 - \left( \frac{y_2}{d_{12}} \right)^2}
\]

### 2.2 The Estimated Offset at \( P_3 \)

Let us first consider the case where we would like to estimate the offset measurement between the point \( P_3 \) and the nylon wire. In this case the coordinates of \( P_3 \) and those of the two anchor points are all known. In order to estimate the offset measurement we will need to know the horizontal vector normal to the nylon wire in the \( OXYZ \) reference system.

Now in the plane \( \text{OXY} \), the unit line vector of the stretched wire, \( \mathbf{L}_2 \), may be found by rotating the line vector \( \mathbf{L}_1 \) through angle \( \theta \),

\[
\mathbf{L}_2 = R_\theta \mathbf{L}_1 = \begin{bmatrix} \cos(\theta) & \sin(\theta) \\ -\sin(\theta) & \cos(\theta) \end{bmatrix} \mathbf{L}_1
\]
A unit vector perpendicular to this line, $\mathbf{N}_2$, is therefore given by,

$$
\mathbf{N}_2 = \begin{bmatrix}
\frac{(\sin(\theta) \, dX_{12} - \cos(\theta) \, dY_{12})}{d_{12}} \\
\frac{(\cos(\theta) \, dX_{12} + \sin(\theta) \, dY_{12})}{d_{12}}
\end{bmatrix}
$$

- (2)

Applying the vector dot product to the vectors $\mathbf{P}_1 \mathbf{P}_3$ and $\mathbf{N}_2$ we see that the estimated offset measurement is given by,

$$
e_3 = (dX_{13} \cdot \mathbf{N}_2 + e_1) - d\text{Cal}_3
$$

where,

$$dX_{13} = \begin{bmatrix} dX_{13} \\ dY_{13} \end{bmatrix}$$

Thus,

$$e_3 = \left\{ \left( \frac{(\sin(\theta) \, dX_{12} - \cos(\theta) \, dY_{12}) \, dX_{13} + (\cos(\theta) \, dX_{12} + \sin(\theta) \, dY_{12}) \, dY_{13}}{d_{12}} \right) + e_1 \right\} - d\text{Cal}_3$$

- (3)

### 2.3 The Reduced Offset at P3

Let us now consider the measured offset from the point $\mathbf{P}_3$ to the nylon wire, and the reduction of this measurement to the line between $\mathbf{P}_1$ and $\mathbf{P}_2$. Again it is assumed that the coordinates of $\mathbf{P}_3$ and those of the two anchor points are all known.

In the system $\text{oxy}$ let,

$$\mathbf{P}_3 = (x_3, y_3), \quad x_3 = \begin{bmatrix} x_3 \\ y_3 \end{bmatrix}$$

We see that,

$$x_3 = dX_{13} \cdot L_2$$

$$y_3 = (d\text{Cal}_3 + e_1) - e_1$$

- (4)  
- (5)
In this same system we see from Figure 1 that the unit vector in the direction of the nylon wire is,

$$
\hat{l}_2 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}
$$

the unit line vector from P₁ to P₂ is,

$$
\hat{l}_1 = \begin{bmatrix} \cos(\theta) \\ \sin(\theta) \end{bmatrix}
$$

and a unit vector perpendicular to \( \hat{l}_1 \) is,

$$
\hat{n}_1 = \begin{bmatrix} -\sin(\theta) \\ \cos(\theta) \end{bmatrix}
$$

We can also see that the reduced offset measurement is given by applying the vector dot product between the vectors \( \vec{x}_3 \) and \( \hat{n}_1 \),

$$
e_3' = \vec{x}_3 \cdot \hat{n}_1
$$

- (6)

$$
e_3' = \left( d\text{Cal}_3 + e_3 \right) \cos(\theta) - \left\{ \left( \cos(\theta) \frac{dX_{12}}{d_{12}} + \sin(\theta) \frac{dY_{12}}{d_{12}} \right) \frac{dX_{13}}{d_{12}} - \left( \sin(\theta) \frac{dX_{12}}{d_{12}} + \cos(\theta) \frac{dY_{12}}{d_{12}} \right) \frac{dY_{13}}{d_{12}} \right\} \sin(\theta)
$$

- (7)