Technical Design Report

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Submitted by HEDgeHOB collaboration
October 2016
Figure on the cover page:

A tiny mechanical watch (left) and its 3.6 GeV proton radiograph (right) recorded during the beam time commissioning of the PRIOR prototype at GSI in 2014 (see Section 2.2). Despite a thick stainless steel case back, the fine details of the interior of the watch are well resolved: the crown and the mainspring, pivots and wheels, jewels, etc.
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1 Executive Summary

Proton radiography or microscopy is a powerful technique for probing the interior of dense objects in dynamic experiments by mono-energetic beams of GeV-energy protons, using a special system of magnetic lenses for imaging and the correction of image aberrations [1]. With this technique, one can measure sub-percent variations in the areal density of thick samples with micrometer spatial resolution and nanosecond temporal resolution. Proton radiography with magnetic lenses was invented in the 1990’s at Los Alamos National Laboratory (LANL) as a diagnostic to study dynamic material properties under extreme pressures, densities and strain rates [2]. Since that time proton radiography and microscopy facilities have also been commissioned at the Institute for Theoretical and Experimental Physics (ITEP) [3, 4] and at the Institute for High Energy Physics (IHEP) [5–7] in Russia.

The capability of radiographic imaging of dynamic systems with unprecedented spatial, temporal and density resolution is of considerable interest for plasma physics and materials research. Therefore high energy proton microscopy (HEPM) is seen as a key diagnostic for high energy density physics experiments with intense heavy ion and proton beams, which are planned at the future Facility for Anti-proton and Ion Research (FAIR) in Darmstadt, Germany [8]. The worldwide unique facility called PRIOR (Proton Microscope for FAIR) will employ high-energy (2 – 5 GeV), high-intensity (up to 2.5 · 10^{13} protons per pulse) proton beams from the SIS-100 synchrotron at FAIR for multidisciplinary research: experiments on fundamental properties of materials in extreme dynamic environments generated by different drivers (pulsed power generators, high-energy lasers, gas guns or explosive-driven generators) prominent for warm dense matter research and high energy density physics as well as the PaNTERA (Proton Therapy and Radiography) experiment [9–12] for biophysics and medicine.

In order to assess the capabilities of the PRIOR facility, a prototype called PRIOR-I has been designed, constructed and successfully commissioned at GSI using 3.6 GeV proton beams [13]. In the design of the PRIOR-I setup NdFeB permanent magnet quadrupoles (PMQ) were used because on the one hand they can provide high field gradients (up to 120 T/m) which are needed to focus GeV-energy protons, and on the other hand they are much less expensive than equivalent electromagnets for the small-aperture lenses. However, already during the first beam time experiments with PRIOR-I in 2014, a continuous degradation of the image quality of the magnifier prototype was observed. This phenomenon was attributed to the radiation damage of the PMQ lenses due to large fluences of spallation neutrons and secondary protons which are mainly produced in the tungsten beam collimator located in a close proximity to the third magnet as well as due to the primary protons scattered to large angles in the target and in the collimator [14–16]. A significant radiation damage of NdFeB PMQs was also observed at LANL [17]. The results of the field distribution measurements after the first PRIOR-I commissioning run have demonstrated that the quadrupole strengths were reduced by 10 – 13% and the high-order field harmonics (relative sextupole, octupole and duodecapole field components) raised to the 1.5–2.5% level. This explains the degradation of the imaging performance of the system.

Since re-magnetizing, re-assembling and re-adjusting of a PMQ-based magnifier would require a huge amount of time and effort as well as special tools, they are not the right choice for a long-term operation of the PRIOR facility at FAIR, where more than two orders of magnitude higher proton beam intensities are expected. Therefore it has been concluded that the final design of the PRIOR proton microscope (PRIOR-II) should employ small but strong and radiation-resistant normal-conducting (NC) electromagnets. The PRIOR-II design has been carefully optimized to achieve nearly the same performance with smaller field gradients provided by NC electromagnets. Following the developed specifications and requirements for proton radiography experiments, the whole HEDgeHOB beam line at FAIR has been redesigned in order to ensure its best possible performance for all of the planned HEDgeHOB experiments: PRIOR, HIHEX and LAPLAS. The final PRIOR design assumes that the PRIOR-II facility will be first fielded at GSI to use protons up to 4 GeV delivered by the SIS-18 synchrotron for static or dynamic experiments.
Later it will be transferred without modifications to the APPA cave at FAIR to use intense proton beams from the SIS-100 synchrotron. The PRIOR facility will provide an image magnification factor of about three at GSI and up to eight at FAIR, with better than 10 µm spatial resolution at the object.

This Technical Design Report is structured as follows. Chapter 2 describes the design of the PRIOR-I prototype and the results of its beam time commissioning at GSI. The experimental requirements and the final ion-optical design of the PRIOR facility is presented in Chapter 5. The design and technical specifications of the PRIOR quadrupole electromagnets are described in detail in Chapter 4. The cost estimates and the project timeline are presented in Chapter ???. Finally, the participants of the project are listed in Chapter 5.
2 Commissioning of the PRIOR-I Prototype

2.1 Design and Construction

The ion-optical design of the PRIOR-I prototype [18] is analogous to the design of the 800 MeV x7 and x3 magnifying lenses at LANL [19,20], but for proton energies up to 4.5 GeV (see Fig. 2.1). The PRIOR-I magnifier employs high-gradient (120 T/m) NdFeB axially and radially segmented permanent magnet quadrupole (PMQ) lenses [21–23] and provides a magnification of about four with a field of view of 15 mm. The illuminating proton beam is matched to the magnifier by five upstream quadrupole electromagnets in order to cancel the second-order position-dependent chromatic aberrations of the system [24].

Figure 2.1: Ion-optical design of the PRIOR-I prototype. Top: matching of the beam to the magnifier (PMQ1-4) using the last five quadrupoles (QDxx, QTxx) of HHT beam line in X- and Y-planes. The MU3 and MU3 elements are the last bending dipole magnets of the beam line. Bottom: imaging by the magnifier. Different colors of the proton trajectories correspond to different scattering angles after passing through the object.

The mechanical design of the magnifier and its PMQ lenses is shown in Fig. 2.2. Four quadrupole magnets in a Russian quadruplet configuration [25] are installed on a common rail. The aperture of the magnets is 30 mm. The outer lenses have a length of 14.4 cm and the inner lenses are twice this length, as required by the beam optics. Each magnet is installed on a motorized support table and can be independently moved along the rail for focusing. Each magnet is made out of 36 mm-long modules (see Fig. 2.2) which can be individually adjusted in x, y and θ directions for aligning the magnetic field axes and the mid-plane tilts. With the help of a precision cylindrical field scanner [15,26,27], the magnetic axes and the field...
2.1 Design and Construction

Figure 2.2: Mechanical design of the PRIOR-I magnifier: four permanent magnet quadrupole (PMQ) lenses are installed on a common rail. A vacuum collimator box is attached to the third lens. Each quadrupole lens (bottom left) contains up to five or ten 36 mm-long modules; each module (bottom right) is built as a double-layer PMQ with 1.8 T pole tip field [23].

Mid-planes of each lens have been aligned to the accuracy of $\pm 20 \mu m$ and $\pm 0.1^\circ$, respectively [15]. Using the same scanner and the field reconstruction procedure [26,27] 3D field maps of the lenses have been obtained. Similar techniques have been applied to obtain an accurate 3D model for the magnetic field which has been used in the ion-optical calculations [28]. A 10 cm long tungsten collimator with an elliptical aperture was installed in the Fourier plane of the magnifier (see Fig. 2.2). In order to adjust the contrast of the proton radiographs for a particular target, collimators with angular acceptances $\theta_c$ of 2, 3 and 4 mrad were used during the commissioning experiments.

Although the PMQ magnifier itself has a length of 1.4 m, a long drift after the magnifier is needed to achieve the required magnification (see Fig. 2.3). Therefore the detector (image collection) system was installed in the newly constructed concrete-shielded detector hall about 9 m downstream of the target location. With a pellicle / mirror arrangement, the system employs two cameras simultaneously: a high resolution (4 Mp) CMOS camera (PCO DIMAX HS) used mainly for static experiments and a fast intensified CCD camera (PCO DICAM PRO) for dynamic measurements. $10 \times 10$ cm columnar CsI and plastic BC-400 scintillators were installed for static and for dynamic measurements, respectively. Preliminary experiments with 800 MeV protons performed at LANL have proven that neither CsI nor plastic scintillators show any image quality or light output degradation even for large irradiation doses ($\approx 10^{11}$ protons per cm$^2$) [15].
2.2 Static Beam Time Commissioning

The PRIOR-I prototype has been commissioned using 3.5 – 4.5 GeV proton beams from the SIS-18 synchrotron of GSI using only moderate intensity ($10^8$ protons per pulse) beams for the static experiments with 3.6 GeV protons. A proton transmission image (radiograph) is obtained as the ratio between the raw images taken with and without an object present under the same proton beam conditions [2]. For this purpose a “beam image” (image without an object) is always recorded shortly before or after imaging an object. The distribution of the areal density is then obtained by applying a transmission – density calibration (see Eq. (2.1) below). Due to the rather low beam intensity and the shot-to-shot beam position fluctuations, in order to enhance the contrast and flatten the background of the radiographs, about 20 – 50 target and beam images were recorded and averaged in the static commissioning experiments [13].

For tuning and measuring the performance of the PRIOR-I microscope, a large set of small static test objects were prepared. The targets were placed in a vacuum target chamber equipped with a precision 6-axis manipulator. The whole setup was evacuated to the $10^{-3}$ mbar pressure in order to maintain radiographic resolution.

The most utilized targets and their proton radiographs are shown in Fig. 2.4. The “fiducial plate” is a 3 mm thick copper plate with 0.5 mm holes machined at 1.5 mm spacing. It was used as a relatively thin target for quick tuning and controlling image distortions as well as for providing spatial calibration while minimizing activation and radiation in the experimental area.

The spatial resolution of the microscope has been measured using the “rolled edge” target — a 20 mm-thick tungsten slab. Two sides of this edge (marked by arrows in Fig. 2.4) are rolled with a 500 mm radius which makes the measurements insensitive to beam-target tilt misalignments of a few milliradian. The spatial resolution can be defined as the standard deviation of the derivative of the measured edge transition (line spread function, LSF; see also the discussion in Sec. 3.2, p. 18). To accurately determine the resolution one also needs to deconvolve the known width of the rolled edge from the measured density profile to get the blur function. However since the extent of the rolled edge itself is only a small contribution ($\sigma_{\text{edge}} \approx 5 \mu m$), for tuning and quick analysis we have fit the edge transition to an error...
function which provides a good estimate of the LSF root mean square (RMS) width. Figure 2.5 shows the horizontal and vertical edge transmission profiles along with the error function fits. These fits resulted in a horizontal resolution of $\sigma_x = 35 \, \mu m$ and a vertical resolution of $\sigma_y = 30 \, \mu m$.

The “Maltese cross” target (Fig. 2.4, right) was used to check the matching conditions \cite{24} required for canceling the position-dependent second-order chromatic aberrations of the microscope. The target is an elongated piece of plastic with Maltese cross like shape and 0.5 mm diameter tungsten wires glued on its back side. When an image of the wires is formed by the magnifier, the protons which were penetrating through both the plastic and the wires have a smaller energy than those which saw only the wires. If the proton beam is not properly matched to the microscope in X- or Y-plane, this difference in the energy loss will result in a slight shifting of the corresponding wires images at the Maltese cross boundaries.

In preparation for dynamic experiments with the PRIOR-I prototype (see Sections 2.3 and 2.4), static mockups of the dynamic targets (exploding tantalum and copper wires) were radiographed to determine how the magnifier could measure the density distribution inside an expanding metallic wire. A proton radiograph of a tantalum wire array is shown in Fig. 2.6. The central wire with 800 $\mu m$ diameter is a mockup of the un-exploded wire for the first dynamic experiments. The transmission profiles across the wires with different diameter show that there is sufficient sensitivity and resolution to measure the areal density of the wire while it expands.

The transmission – target thickness dependency, $T(z)$ of a proton radiography system can be described by a simple analytic model to a percent accuracy \cite{29}:

$$T(z) = \frac{e^{-z/\lambda}}{T(0)} \left( 1 - e^{-\frac{\theta^2(z) + \beta^2 s^2}{2}} \right), \quad (2.1a)$$

$$\theta(z) = \frac{13.6 \, MeV}{\beta pc} \sqrt{\frac{z}{X_o}} \left[ 1 + 0.088 \log_{10} \left( \frac{z}{X_o} \right) \right]. \quad (2.1b)$$

The first term $e^{-z/\lambda}$ in Eq. (2.1a) describes the removal of the protons due to nuclear interactions in the target material with the nuclear collision length $\lambda$. The second term is due to the multiple Coulomb...
2.2 Static Beam Time Commissioning

Figure 2.5: Spatial resolution measurements with rolled edge. Measured horizontal (blue) and vertical (green) edge transitions near the center of the image are shown along with the corresponding error function fits. The horizontal and vertical RMS widths of this edge were measured to be 35 µm and 30 µm, respectively.

Figure 2.6: Tantalum wire array (left), its PRIOR-I radiograph with 3.6 GeV protons (middle) and a zoomed central part of the middle image (right) with a transmission profile across the wires (red curve).

Scattering: \( \theta(z) \) is the RMS scattering angle and \( \theta_c \) is the angular acceptance of the magnifier defined by its collimator. Assuming a normal distribution of the multiple scattering, the \( \theta(z) \) dependency can be approximated with sufficient accuracy by the Molière theory [30], Eq. (2.1b). Here \( p \) is the proton momentum, \( \beta \) is the proton velocity in units of the velocity of light \( c \), and \( X_o \) is the radiation length of the material. The only empirical parameter in the model Eq. (2.1) is the angular spread of the beam \( \phi \) due to the beam emittance and overburden material (e.g. vacuum windows or air) downstream of the target which is added in quadrature to the multiple scattering angle \( \theta(z) \) caused by the object, and describes a small attenuation of the beam when there is no object.

To test the density sensitivity of the microscope and to obtain the transmission – target thickness calibration \( T(z) \) needed for dynamic experiments, radiographs of a series of step wedge targets were collected (see Fig. 2.7). Identical tantalum and copper step wedges were used. In order to replicate the conditions of the dynamic commissioning, the step wedge targets were placed in the middle of the dynamic explosion chamber (see sec. 2.3, Fig. 2.10) which was filled with water or left in air. Two collimators with \( \theta_c \) equal to 2 and 3 mrad were used. Figure 2.7 shows that the measured transmission is in good agreement with
2.2 Static Beam Time Commissioning

Figure 2.7: Transmission – areal density calibration with step wedges. The tantalum step wedge target is shown in the left inset and the proton radiographs of the identical copper and tantalum step wedges (0.56, 2.06, 4.07 and 6.05 mm step thicknesses) are shown in the right insets. The transmission data measured with different collimator acceptance angles $\theta_c$ for the targets placed in air and in water (30 mm thickness, see sec. 2.3) is shown along with the model Eq. (2.1) (solid lines) which has only one fitting parameter — the angular spread of the beam, $\phi$ (see explanations in the text).

The data demonstrates a remarkable density sensitivity and proves that the PRIOR-I prototype can be used for radiographic density measurements.

Experience with proton radiography at the pRad facility at LANL has proven that radiographing various static objects is a good way to test a new radiography configuration. Extracting the geometry of challenging objects can often test the ability to resolve detailed structure of the objects, providing an opportunity to study and image beyond rolled edges, step wedges and other calibration targets. To fill this role a few “common” objects were radiographed for this effort, and two of them are shown in Fig. 2.8: small quartz and mechanical watches. The slight non-flatness of the radiographic background is due to the data averaging and shot-to-shot beam position fluctuations. The obtained proton radiographs of these objects with complex interior structures clearly demonstrate the remarkable radiographic capabilities of the PRIOR setup.

As an unfortunate result of the first PRIOR-I run, we have observed a continuous degradation of the image quality and spatial resolution towards the end of the experiment. This phenomenon was attributed to the radiation damage of the PMQ lenses due to large fluences of spallation neutrons which are mainly produced in the tungsten beam collimator located in a close proximity to the third magnet (Fig. 2.2 p. 5) as well as due to the primary protons scattered to large angles in the target and in the collimator. A significant radiation damage of neodymium-iron-boron PMQs has been also observed at LANL [17]. Because of this the 3D fields maps of all the magnets were measured after the first commissioning run. The results of the field distribution measurements have demonstrated a significant damage of the PMQs, especially of the first and the third lenses: the quadrupole strengths were reduced by 10 – 13 % and the high-order field harmonics (relative sextupole, octupole and duodecapole field components) raised to the 1.5 – 2.5 % level. This explains the degradation of the imaging performance of the system. We have also performed additional simulations and measurements of the PMQ radiation damage phenomenon [14,15,31] which confirm the results of the LANL study [17]. Unfortunately, the time between the static and dynamic PRIOR-I commissioning runs was not sufficient for re-magnetizing, reassembling and readjusting the lenses and we had to use the microscope in the dynamic experiments (see sec. 2.4) in the same suboptimal
2.3 Underwater Electrical Wire Explosion Experiment

Underwater electrical wire explosion (UEWE) is an efficient method for creating and studying warm dense matter in the laboratory [32,33]. The main advantages of the UEWE are the absence of the parasitic plasma formation along the wire surface due to high electric breakdown threshold (>300 kV/cm) of the water and relatively small wire expansion velocity ($10^5$ cm/s). These features allow to retain high current densities in the wire (up to $10^9$ A/cm$^2$) and therefore, by using a moderate pulsed power generator, one can create dense strongly coupled plasmas characterized by 10 – 100 kJ/g specific energy, near-solid density state.

Figure 2.8: PRIOR-I proton radiographs (3.6 GeV protons) of complex targets. Top: a quartz watch and a radiograph of its central part. One can clearly see the battery and movement. Having a sufficient contrast, one can also see the hour, minute and even second hands of the watch. Bottom: a tiny mechanical watch. Despite a thick stainless steel case back, the fine details of the interior of the watch are well resolved: the crown and the mainspring, pivots and wheels, jewels, etc.
2.3 Underwater Electrical Wire Explosion Experiment

Figure 2.9: Pulsed power setup for underwater electric wire explosion experiments (UEWE) installed at the HHT area of GSI. A water filled UEWE explosion target chamber surrounded by four pulsed power generators is placed in front of the PRIOR-I proton microscope.

and several eV temperature. The main challenge in warm dense matter experiments is the determination of plasma parameters, and especially temporally and spatially resolved measurements of the target density. High energy proton microscopy is a unique diagnostic technique to address this problem.

Figure 2.10: Scheme of the UEWE explosion chamber and target diagnostics. A thin exploding wire (red) in the center of the chamber is illuminated from one side by a proton beam for HEPM measurements, and from the other side by a laser diode backlighter for optical diagnostics.
A new pulsed power UEWE setup has been constructed for dynamic commissioning of the PRIOR-I prototype (Fig. 2.9). The pulsed power generator (10 μF, up to 50 kV charging voltage and 12.5 kJ stored energy) consists of four modules and can drive currents of about 200 kA in amplitude and 1.8 μs rise time through a load at charging voltage of 35 – 40 kV.

During the PRIOR-I experiments, tantalum wires (0.8 mm diameter and 40 – 50 mm length) were quickly heated by a pulsed current to dense plasma conditions characterized by specific enthalpy level about 5 – 15 kJ/g and ~ km/s expansion velocities. Tantalum has been chosen for the experiments due to its high density which allows for a higher contrast of proton radiographs.

The construction of the UEWE chamber and the scheme of the target diagnostics is shown in Fig. 2.10. A wire is placed in the middle of the 11 cm diameter stainless steel explosion chamber which is filled with deionized water. A special effort has been taken to design water shock dampers in order to minimize the amount of material needed to separate the water-filled UEWE chamber and the vacuum PRIOR beam line. The dampers (see Fig. 2.10) are 18.5 cm-long, 6.6 cm-diameter aluminum pipes containing from nine to twelve 150 μm-thick Mylar foil stacks in holders with 22 mm opening. A shock wave induced in water by an exploding wire consequently breaks the Mylar foils filling the damper pipe with water until it is completely stopped before the vacuum window of the beam line. From the explosion chamber side, the dampers are equipped with plastic insets sealed by a thin rubber. The insets allowed the reduction of the water layer thickness in the proton beam direction down to 30 mm.

![Figure 2.11: Electrical measurements in the UEWE experiment. Measured waveforms of the current I (a) and voltage U (b) are shown along with wire resistance R (c), resistive voltage \( U_r = I \cdot R \) (d), deposited power \( P_r = I^2 \cdot R \) (e) and energy \( E_r = \int_0^t P_r(\tau) d\tau \) (f).](image)

The current flowing through a wire \( I(t) \) and the voltage drop on the load \( U(t) \) were measured by a Rogowski coil and a resistive voltage divider, respectively. The resistance of the wire \( R(t) \) can be obtained from the following equation:

\[
U(t) = I(t) \cdot R(t) + L(t) \frac{dI(t)}{dt} + I(t) \frac{dL(t)}{dt}.
\] (2.2)

The last term in Eq. (2.2) gives a small correction at the 5% level which can be applied if the wire radius \( r(t) \) is known, e.g. from optical or radiographic measurements: \( dL/dt \approx \mu_0 \ell / 2\pi \cdot r'(t)/r(t) \), where \( \ell \) is the length of the wire and \( r'(t) \) is its radial expansion velocity. Eq. (2.2) can also be used in its integral form (equation for the energy balance), and under the assumption of a constant load inductance.
2.3 Underwater Electrical Wire Explosion Experiment

$L(t) = L_0$, the resistance of the exploding wire $R(t)$ can be obtained by solving the following equation:

\[
\int_0^t I^2(\tau)R(\tau)d\tau = \int_0^t I(\tau)U(\tau)d\tau - \frac{1}{2}L_0I^2(t). \tag{2.3}
\]

This integral equation was solved by forward substitution with subsequent smoothing of the result. The value of the load inductance $L_0$ was adjusted while solving Eq. (2.3) so that the resistivity of the wire in the beginning of the discharge is equal to the known resistivity of solid tantalum \[35\]. The results of electrical measurements of a typical UEWE experiment with PRIOR-I is shown in Fig. [2.11]. The plateau on the resistance signal at about 0.8 $\mu$s corresponding to the measured enthalpy of 0.5 – 1 kJ/g can be attributed to the melting transition and the following rapid heating of expanding liquid tantalum. The quick rise of the resistance at 1.1 – 1.4 $\mu$s and an enthalpy in the target material of about 4 kJ/g may indicate the onset of a rapid evaporation.

In addition to the HEPM measurements with the PRIOR-I prototype (sec. [2.4]) and electrical measurements, an optical setup has been installed for target diagnostics. The optical diagnostics of the exploding wires (backlighting and thermal emission imaging) allows to determine the radius of the discharge channel as well as the velocity of the shock wave induced in water. The setup consists of a 450-nm CWL, 4-W fiber-coupled laser diode backlighter, two fast intensified CCD cameras (PCO DICAM PRO), a streak camera (HAMAMATSU C10910) and a set of lenses, mirrors, filters and beam splitters (see Fig. [2.10]).

Figure 2.12: Optical diagnostics of the UEWE experiment (see explanations in the text). Left: streak camera record of exploding tantalum wire. Right: CCD camera images of the same wire at $t = 0$ (top) and $t = 1.95$ $\mu$s (bottom).

A typical streak camera record of an exploding tantalum wire with initial radius of 0.4 mm is shown in Fig. [2.12] left. Shortly after the beginning of the discharge ($t = 0$) a sound wave ($v \approx 1.5$ km/s) is launched in water by the rapidly heated and expanding wire. At the moment close to the wire explosion (fast boiling), a shock wave ($v \approx 2.3$ km/s corresponding to the pressure $P \approx 9.3$ kbar) is also launched and the thermal self-emission of the tantalum plasma becomes visible. At later times $t > 4$ $\mu$s, the discharge channel radius can be traced again by the shadowgraphy. The shock wave and the wire discharge channel were also clearly visible in the CCD images (Fig. [2.12] right).
2.4 Dynamic Beam Time Commissioning

After the static beam time commissioning of the PRIOR-I setup and off-line tests of the UEWE setup, an experimental campaign of dynamic experiments with the PRIOR-I microscope took place at GSI. In comparison with the previous run, the proton beam intensity was increased by more than two orders of magnitude (up to $10^{11}$ protons per pulse) and a new beam diagnostics for high energy protons (scintillator screens and cameras) were integrated into the HHT beam line to ensure proper beam alignment and matching. Unfortunately, the shot-to-shot variations of the beam position and intensity distribution which were observed during the commissioning run with static experiments remained. The 3.6 GeV proton beam has been delivered in four $\approx 40$ ns long bunches with about 150 ns inter-bunch spacing. Since the PRIOR image collection system was equipped with one fast camera, only one out of four bunches was used for the dynamic experiments.

Before the experiments with dynamic objects, a series of tests to determine the achievable temporal resolution of the microscope was conducted using a plastic scintillator (BC-400, decay time 2.4 ns) and an intensified CCD camera (PCO DICAM PRO, fast shutter down to 3 ns) in the image collection system. It has been shown that with the available proton beam intensity, one can achieve a 5 – 10 ns temporal resolution without significant deterioration of the imaging properties by gating the detector while using one of the four bunches. Due to the shot-to-shot instability of the beam, a 20 ns detector gate was used for the dynamic target experiments.

The dynamic PRIOR-I commissioning was carried out using the developed UEWE setup and 0.8 mm diameter tantalum wires. In total, about twelve successful dynamic shots with the PRIOR-I setup were completed. In these shots we have varied the power deposited in the wires by changing the wire length and capacitor charging voltage as well as the timing of the proton radiographs.

![Figure 2.13: Processing of the dynamic proton radiographs.](image)

Because of the shot-to-shot fluctuations of the beam position during the PRIOR-I dynamic commissioning run, the beam images could not be directly used for the processing of the dynamic radiographic data, and the information about the transverse beam intensity distribution had to be deduced from the dynamic target images themselves. The analysis of a large number of the recorded beam images has shown that the intensity distribution in the central area of the beam can be well approximated in each shot by an asymmetric Gaussian function. Using this knowledge, the transmission images were obtained by dividing the raw images by the empirical beam intensity distribution function with the function parameters fitted to the the same radiograph using the image areas which are not occupied by a target. An example of such data processing is shown in Fig. 2.13 for “static” (taken before shot) and “dynamic” (taken during wire...
explosion) proton radiographs of a UEWE experiment.

Figure 2.14: Radii of exploding tantalum wires measured with PRIOR-I prototype (symbols) and by optical streak diagnostics (solid lines, see also Fig. 2.12 left). The initial radius of the wires was 0.4 mm. The data is shown for four shots with different levels of the peak specific power deposition from 4.8 to 9.4 GW/g.

Although the ultimate goal of the UEWE experiments with the PRIOR-I microscope was to measure the radial density distribution of an exploding wire, the significant degradation of the spatial resolution due to the radiation damage of the PRIOR-I PMQ lenses as well as potential imperfections of the non-standard data processing procedure did not allow achieving this goal with a sufficient accuracy. Nonetheless, it was possible to use the obtained radiographic data for determining the radii of the exploding wires by deconvolving a Gaussian resolution blur and using the static wire radiographs as a reference. A comparison of these radiographic results with the results of the optical diagnostics (see sec. 2.3) is shown in Fig. 2.14 for four shots with different levels of the peak specific power deposition and correspondingly—different expansion velocities. The error bars of the radiographic results are caused mostly by the possible uncertainties due to the data processing procedure (deconvolution of the blur and the transmission data accuracy). Despite relatively large error bars, one can see that the obtained PRIOR-I proton radiographic results are in reasonable agreement with the optical measurements for all dynamic UEWE experiments.

2.5 Conclusions

As a result of a joint international effort, a prototype of the PRIOR high energy proton microscopy facility (PRIOR-I) has been designed, constructed and successfully commissioned at GSI. The beam time commissioning using intense 3.5 – 4.5 GeV proton beams delivered by the SIS-18 synchrotron has demonstrated 30 μm spatial and 10 ns temporal resolutions with a remarkable density sensitivity. For the dynamic commissioning of the PRIOR-I magnifier, a new pulsed power setup for studying properties of matter under extremes has been developed and is operational at GSI. The commissioning experiments have indicated that neodymium-iron-boron PMQ lenses are not an appropriate choice for the final design of the PRIOR facility which will be used at FAIR with high-energy and high-intensity proton beams. The reason for is the severe radiation damage of the PQM magnets. Although samarium-cobalt permanent magnets are known to be more radiation-tolerant [17], they are also not the right choice for a long-term operation of the PRIOR facility at FAIR, where more than two orders of magnitude higher proton beam intensities are expected.

Therefore the final design of the PRIOR proton microscope (see Chapter 3) which is called PRIOR-II employs small but strong and radiation-resistant electromagnets (60 mm aperture and 1.3 T pole tip field).
3 Requirements and Ion Optical Design

This chapter describes the requirements, ion optical design and performance of the new PRIOR-II proton microscope. The design assumes that the PRIOR-II setup will be first fielded at the HHT area of GSI to use protons up to 4 GeV delivered by the SIS-18 synchrotron for static or dynamic experiments, and later will be transferred without modifications to a new experimental area at FAIR to use intense 2 – 5 GeV proton beams of the SIS-100 synchrotron. The PRIOR-II facility will provide a magnification of about 3.5 at GSI and up to 8 at FAIR with < 10 µm spatial resolution at the object.

3.1 Experimental Requirements

The main characteristics of a proton microscope for an experiment are the spatial resolution (\(R\)), field of view (FOV), image contrast and achievable density reconstruction accuracy. All these parameters are inter-connected, and for a particular microscope design depend on the initial proton energy and on the thickness of an object. The FOV of a proton microscope is typically 1/2 – 2/3 of the aperture of the quadrupoles. The FOV shall on the one hand be sufficiently large for foreseen experiments but on the other hand too large quadrupole apertures may significantly reduce the spatial resolution due to reduced field gradients. Based on experience obtained from the PRIOR-I experiments and from previous experiments with pRad X1 and X3 systems at LANL, the quadrupole pole tip radius of 30 mm has been chosen under assumption that the magnets are able to deliver a pole-tip field of up to 1.3 T. This shall give a sufficient gradient. Based on experience obtained from the PRIOR-I experiments and from previous experiments with pRad X1 and X3 systems at LANL, the quadrupole pole tip radius of 30 mm has been chosen under assumption that the magnets are able to deliver a pole-tip field of up to 1.3 T. This shall give a sufficient FOV and a reasonably good (< 10 µm) spatial resolution.

Generally, the choice of the initial proton energy, \(E_0\), depends on the particular experiment. In many cases the spatial resolution can be well approximated by the chromatic RMS resolution defined by Eq. (3.17), p. [21] \[ CR \propto \theta_c \cdot \sigma_k \] where \(\theta_c\) is the collimator acceptance and \(\sigma_k\) is the relative energy spread of the beam after the object (see sec. 3.2.2 below and Figs. 3.3 and 3.4 for the explanation). Qualitatively, for a thin object and relatively high \(E_0\), the \(\sigma_k\) will be small and therefore the chromatic resolution can be reduced. However, the amount of scattering in the object will also be reduced thus decreasing the contrast of the image (unless a very narrow collimator \(\theta_c < 1 - 2\) mrad is used). On the other hand, if the object is thick while \(E_0\) is too low, the spatial resolution can become inadequately poor. Therefore for each particular object (or experiment) with a certain thickness range to be radiographed there’s an optimum \(E_0\) providing a good image contrast within the necessary areal density region and a sufficient spatial resolution.

The best radiographic contrast and the density reconstruction accuracy are achieved when the transmission of the microscope \(T(z) \approx 40 - 60\%\), where \(z\) is the thickness of the object. This happens when the collimator acceptance \(\theta_c\) is matched to the RMS scattering angle in the object, \(\theta_c \approx \theta(z)\) (see Eq. (2.1), p. [7] and Fig. 2.7, p. [9]). If the collimator (\(\theta_c\)) is chosen so that for a reference target thickness, \(z_o : T(z_o) = 50\%\), the dynamic range for the thickness variation, \([z_{\text{min}} : z_{\text{max}}]\) can be defined as e.g., \(T(z_{\text{min}}) \approx 80\%\) and \(T(z_{\text{max}}) \approx 20\%\). Now for a given \(E_0\) one can determine the acceptance of the matched collimator \(\theta_c : T(z_o) = 50\%\), estimate the dynamic range and the corresponding chromatic resolutions. The above described physics performance optimization procedure is quantitatively demonstrated in Fig. 3.1 for a tantalum target. For example, let’s choose the reference object thickness \(z_o = 2\) mm and \(E_0 = 5\) GeV. The closest to this point 50% transmission contour line (red) correspond to the \(\theta_c = 2\) mrad collimator. The black contour lines at the same point indicate \(\sigma_k = 0.65 \cdot 10^{-3}\). The other two thick red lines give the thickness dynamic range of [0.8 : 6] mm at 80/20% transmission levels. Now using Eq. (3.17) and assuming the average chromatic length of 3.8 m, one can calculate \(CR \approx 0.479 \times 3.8 \times 2 \times 0.65 = 2.4\) µm. Analogous results for different \(E_0\) obtained from Fig. 3.1 are shown in Table 3.1.

One can see that for this energy range the energy spread of the beam after the target is mostly defined...
3.1 Experimental Requirements

Figure 3.1: Scattering and energy loss straggling in a tantalum target as a function of proton energy \(E_0\) and the target thickness \(z\). The colored thick lines with labels \(\theta_c\) values in mrad are the contour lines of \(T = 50\%\) transmission for the corresponding collimators. Two additional thick lines of the same color below and above show the corresponding \(T = 80\%\) and \(T = 20\%\) transmission levels, respectively. The thin colored lines show the other transmission levels in 10\% steps. The black thin lines are the contour lines of the relative energy spread after the object \(\sigma_k\). The black labels give the corresponding \(\sigma_k\) values in \(10^{-3}\) units.

By the initial energy spread \((5 \cdot 10^{-4})\) whereas the energy loss straggling in the target plays only a minor role. Therefore the chromatic resolution is defined by the collimator acceptance, \(\theta_c\) which provides the required transmission level. It may seem that by increasing \(E_0\) from 5 GeV to 10 GeV and correspondingly, switching from 2 mrad to 1 mrad collimator one could still enhance the resolution. However with a limited maximum field gradient, a microscope designed for 10 GeV will have a larger chromatic length than for 5 GeV and this will reduce the chromatic resolution. Moreover the 1 mrad collimator for a typical PRIOR-II design will have an elliptical opening of about \(2.5 \times 4.5\) mm only and the length of 10 cm which will bring in additional alignment issues.

Table 3.1: Performance of a proton microscope (chromatic length 3.8 m) as a function of initial proton energy \(E_0\) for a tantalum target with \(z_o \approx 2\) mm (see Fig. 3.1 and explanations in the text).

<table>
<thead>
<tr>
<th>(E_0), GeV</th>
<th>(\theta_c) ((T = 50%)), mrad</th>
<th>(\sigma_k), (10^{-3})</th>
<th>(CR), (\mu m)</th>
<th>(z_{min} : z_{max}), mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>5</td>
<td>10</td>
<td>0.9:6</td>
</tr>
<tr>
<td>(\theta_c) ((T = 50%)), mrad</td>
<td>8</td>
<td>5</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>(\theta_c) ((T = 50%)), mrad</td>
<td>0.95</td>
<td>0.78</td>
<td>0.65</td>
<td>0.61</td>
</tr>
<tr>
<td>(\theta_c) ((T = 50%)), mrad</td>
<td>14</td>
<td>7.1</td>
<td>2.4</td>
<td>1.1</td>
</tr>
<tr>
<td>(\theta_c) ((T = 50%)), mrad</td>
<td>0.9:6</td>
<td>1.7</td>
<td>0.8:6</td>
<td>0.7:5</td>
</tr>
</tbody>
</table>

After analyzing in the same way many different configurations of the proposed PRIOR experiments including the PaNTERA project [12], it can be concluded that the most useful proton energy range for the PRIOR-II design is 2-5 GeV. There are no such thick objects (high-Z and much thicker than FOV sizes) foreseen which would justify for choosing \(E_0 > 4 \text{ – 5 GeV}\). Reducing the proton energy below 1 GeV could make sense only for very thin objects. In this case however it may be more efficient to use an inverted collimator [36][37].
3.2 Determination of Spatial Resolution

The performance of an imaging optical system can be fully characterized by its point spread function, \( \text{PSF}^{0,0}(x, y) \) and its Fourier transform — optical transfer function, \( \text{OTF}^{0,0}(\nu_x, \nu_y) \) where \( \nu_x, \nu_y \) are the spatial frequencies measured in e.g., lp/mm. If a system does not have strong position-dependent aberrations, an on-axis point-spread function, \( \text{PSF}^{0,0}(x, y) \) is sufficient to characterize its behavior. The absolute value of the OTF is called modulation transfer function, \( \text{MTF}(\nu) \) which is usually normalized so that \( \text{MTF}(0) \equiv 1 \). The MTF gives the relative contrast of an image as a function of the spatial frequency. Therefore one of many existing definitions of the optical system resolution is the point \( 1/\nu_{\text{max}} \) where the contrast is reduced to zero \( \text{MTF}(\nu_{\text{max}}) = 0 \) or to a given low value.

Figure 3.2: On-axis \( \text{PSF}^{0,0}(x, y) \) (left) and off-axis \( \text{PSF}^{10mm,10mm}(x, y) \) (right) with the corresponding x- and y-projections (LSFs) in the object plane calculated for one of the PRIOR designs using 3rd-order transfer map and particle tracing by the PROSIT code [38]. The "star" pattern of the on-axis PSF is a typical signature of the spherical aberration. The off-axis PSF has a much more complex and asymmetric structure.

However it is very difficult to measure and to calculate PSFs. The calculation of a PSF is computationally expensive because it requires tracing a large number of particles (or rays) through the system. An example of the calculated PSFs for PRIOR is shown in Fig. 3.2. In practice, optical systems are often characterized by the projections of the PSF to \( x \)- and \( y \)-axes, the marginal PSFs which are equivalent to the line spread functions \( \text{LSF}_x(x) \), \( \text{LSF}_y(y) \). The edge spread function (ESF, the integral of the LSF) is the most convenient to be obtained in an experiment by measuring transmission profiles across a sharp edge (see Fig. 2.5, p. 8).

The spatial resolution can be defined as the standard deviation of the derivative of a measured edge transition which is the root mean square (RMS) width of the LSF, or

\[
R_x^2 \approx \langle X^2 \rangle = \int X^2 \text{LSF}_x(X) dX.
\]

We will call \( R_x \) and similar quantities the RMS resolution of a proton imaging system.

3.2.1 Analytical Calculation of RMS Resolution

Let the coordinates of a particle at the object plane of an imaging system be \( x = \{x_1, x_2, \ldots, x_6\} \), where according to usual (e.g., COSY INFINITY code [39,40]) notation

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\[ x_1 = x, \quad x_2 = p_x / p_0, \]
\[ x_3 = y, \quad x_4 = p_y / p_0, \]
\[ x_5 = l = \frac{(t - t_0)v_0\gamma}{1 + \gamma}, \quad x_6 = \frac{K - K_0}{K_0}, \]  
(3.2)

where \( p_x \) and \( p_y \) are the horizontal and vertical momentum components, \( p_0, K_0, v_0, t_0 \) and \( \gamma \) are the momentum, kinetic energy, velocity, time of flight, and total energy over \( m_0c^2 \) of the reference particle, respectively. These coordinates are canonical. The initial phase space distribution of the beam is given by the probability density function (p.d.f.), \( f(x) \). After passing through the system, the coordinates of a particle at the detector (image) plane are \( X = \{X_1, X_2, \ldots, X_6\} \) and the p.d.f. of the beam is \( F(X) \). The final coordinates, \( X \) are related to initial coordinates, \( x \) by a transfer map (Taylor expansion) \( M = \{M_1, M_2, \ldots, M_6\} \):

\[ X_k = M_k \circ x = \sum_{\{i_k\}, \sum i_k = N} M_{k, \{i_k\}} x_1^{i_1} x_2^{i_2} \cdots x_6^{i_6} \]  
(3.3)

where \( N \) is the order of the transfer map. The elements \( M_{k, \{i_k\}} \) of the vectors \( M_k \) are proportional to the partial derivatives of the final coordinate \( X_k \) with respect to the corresponding initial coordinates \( \{x_i\} \). E.g., the second order element \( M_{1, \{2,6\}} \propto \frac{\partial X_1}{\partial x_2} \delta \). In our earlier paper [41] we have developed a method for the analytical calculation of any \( m^{th} \) power moment of the final variable \( X_k \). This can be done if the moments of the initial p.d.f. up to power \( m \cdot N \) and the transfer map \( M_k \) are known:

\[ \langle X_k^m \rangle = \iiint (X_k)^m \cdot F(X) \, dX \]
\[ = \iiint (M_k \circ x)^m \cdot f(x) \, dx \]
\[ = \iiint \left( \sum_{\{i_k\}, \sum i_k = N} M_{k, \{i_k\}} x_1^{i_1} \cdots x_6^{i_6} \right)^m f(x) \, dx \]
\[ = \sum_{\{i_k\}, \sum i_k = m \cdot N} m! C_{k, \{i_k\}} \int \cdots \int x_1^{i_1} \cdots x_6^{i_6} f(x) \, dx \]  
(3.4)

where \( \langle x_1^{i_1} \cdots x_6^{i_6} \rangle \) are the moments of the initial distribution \( f \) and the coefficients \( m! C_{k, \{i_k\}} \) are the products of \( m \) elements of the vector \( M_k \). In that paper we have used the theory of moments to develop a statistical approach for the beam shaping (modification of the charged particle beam intensity distribution in the phase space using nonlinear ion-optical elements). Here we apply it for the analytical calculation of the second moments of the LSF\(^{X,0}_{x,0}\) and LSF\(^{Y,0}_{y,0}\), i.e. — for determination of the microscope RMS resolution.

Let’s consider a special case of the initial p.d.f.,

\[ f(x) = \delta(x_1 - x_0) \delta(x_3 - y_0) f_{x_2 x_4 x_5 x_6}(x_2, x_4, x_5, x_6). \]

(3.5)

Here all the particles are coming from a single point \( (x_0, y_0) \) at the object plane. Obviously, the corresponding marginal distributions at the image plane,

\[ F_{X_1 X_3}(X_1, X_3) \equiv \text{PSF}(X, Y) \]
\[ F_{X_1}(X_1) \equiv \text{LSF}_X(X), \quad F_{X_3}(X_3) \equiv \text{LSF}_Y(Y) \]

(3.6)

will be the point spread and line spread functions of the optical system. Furthermore, as for calculating the imaging properties of a system we are not interested in the time of flight, we can assume \( f_{x_5}(x_5) \equiv 0 \). In most cases one can also assume for a PSF calculation that \( f_{x_2 x_4 x_6}(x_2, x_4, x_6) = f_{x_2 x_4}(x_2, x_4) \cdot f_{x_6}(x_6) \), although it is not a necessary assumption. Then using variables Eq. (3.2), Eq. (3.5) can be rewritten as:

\[ f_{\text{PSF}}(x) = \delta(x - x_0) \delta(y - y_0) f_{a b}(a, b) f_k(k). \]

(3.7)
The energy distribution of the proton beam after an object is normal with its width, $\sigma_k$ due to the initial energy spread of the beam and the energy loss straggling in the object:

$$nf(k) = \frac{1}{\sigma_k \sqrt{2\pi}} \exp\left( -\frac{k^2}{2\sigma_k^2} \right). \quad (3.8)$$

The angular distribution is also normal with the dispersion, $\sigma_a = \sigma_b$ given mostly by the multiple Coulomb scattering in the object. The collimator action — the angular cut at $\theta_c = \sigma_a$ level (i.e. — the collimator acceptance is matched to the object thickness) can be well described by choosing the corresponding initial angular distribution — circularly truncated normal distribution:

$$2D f(a, b) = \begin{cases} \frac{1}{2\pi \left(1 - \frac{1}{\sqrt{e}}\right)\sigma_a^2} \exp\left( -\frac{a^2 + b^2}{2\sigma_a^2} \right) & \text{if } (a^2 + b^2) < \sigma_a^2 \\ 0 & \text{otherwise.} \end{cases} \quad (3.9)$$

In the last equation we implicitly redefined the coordinates $x_2 = a$ and $x_4 = b$ to be the mechanical momenta, $a = p_x/p_z$ and $b = p_y/p_z$ instead of the canonical momenta used in Eq. (3.2).

It is possible to calculate any moment of the truncated normal distribution Eq. (3.9) analytically:

$$2D H_{2m,2n} = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} a^{2m} b^{2n} \cdot 2D f(a, b) dadb = \frac{\sigma_a^{2(m+n)}}{2\pi \left(1 - \frac{1}{\sqrt{e}}\right)} \left(\cos \theta \right)^{2m} \left(\sin \theta \right)^{2n} d\theta \int_{0}^{1} x^{2(m+n)+1} \exp\left(-\frac{x^2}{2}\right) dx \quad (3.10)$$

$$= \frac{\sigma_a^{2(m+n)}}{\pi \left(1 - \frac{1}{\sqrt{e}}\right)} \frac{\Gamma\left(m + \frac{1}{2}\right)}{\Gamma\left(m + n + 1\right)} \frac{\Gamma\left(n + \frac{1}{2}\right)}{\Gamma\left(m + n + 1\right)} 2^{m+n} \gamma\left(m + n + 1, \frac{1}{2}\right),$$

where $\gamma(x, a)$ is the incomplete gamma function, $\gamma(x, y) = \Gamma(x) - \Gamma(x, y)$. The moments of the normal distribution Eq. (3.8) are simply

$$^n \mu_{2m} = (2m - 1)!! \sigma_k^{2m}. \quad (3.11)$$

With a certain loss of accuracy ($10$–$20\%$), one can simplify the moments calculations by assuming $\nu^{2D} f(a, b) = 2D f(a) \cdot 1D f(b)$, where

$$1D f(a) = \begin{cases} \frac{1}{\sigma \sqrt{2\pi} \text{erf}(1/\sqrt{2})} \exp\left( -\frac{a^2}{2\sigma^2} \right) & \text{if } |a| < \sigma_a \\ 0 & \text{otherwise,} \end{cases} \quad (3.12)$$

is the 1D truncated normal distribution with its moments

$$1D H_{2m} = \frac{\sigma_a^{2m} \gamma^{m + \frac{1}{2}}}{\sqrt{2\pi} \text{erf}(1/\sqrt{2})} \gamma\left(m + \frac{1}{2}, \frac{1}{2}\right) \quad \gamma\left(m + \frac{1}{2}, \frac{1}{2}\right)$$

$$= \frac{\sigma_a^{2m}}{\text{erf}(1/\sqrt{2})} \sqrt{\frac{2}{\pi e}} \sum_{i=m}^{\infty} (2m - 1)!!.$$ (3.13)

This is equivalent to assuming a rectangular angular cut (a "rectangular" collimator) and would give a slightly conservative resolution estimate. Furthermore, for a calculation of the second moment, one
can also approximate the truncated normal angular distribution Eq. (3.12) by a parabolic or a uniform distributions with the corresponding dispersions.

With the above knowledge, the RMS spatial resolution of a microscope at the object plane can be calculated simply as

\[ R_{x}^{0,0} = \frac{\sqrt{\langle X^2 \rangle}}{M}, \quad R_{y}^{0,0} = \frac{\sqrt{\langle Y^2 \rangle}}{M}, \]  

(3.14)

where the "PSF" initial phase space distribution Eq. (3.7) shall be used for calculating the moments and \( M \) is the magnification of the system.

We will use for the RMS resolution the symbols \( R^{0,0} = AR \) and \( R^{x\theta,y\theta} = OR \), when the RMS resolutions are calculated as on-axis and off-axis LSFs dispersions, respectively. Note that in the initial proton beam distributions \( f_{xa}(x, a) \) and \( f_{yb}(y, b) \) the covariances \( \langle xa \rangle \) and \( \langle yb \rangle \) are not equal to zero. On the contrary, the values of these covariances are defined by the beam matching conditions (see Eq. (3.16) below). Therefore while calculating the \( OR_{x,y} \) resolutions one has to shift the angular distribution Eq. (3.9) so that it will have the means \( a_0 \) and \( b_0 \) proportional to the LSF point coordinates \( x_0 \) and \( y_0 \). The analytical procedure described above to calculate the RMS resolution has also been verified by numerical ray tracing.

There is another way to estimate the resolution using the moments. In an ideal magnifier with magnification \( M \) every point of the object plane should be mapped to a single point in the image: \( X = M \cdot x \) and \( Y = M \cdot y \). Therefore one can obtain an "integral" resolution by calculating the second moment of the quantity \( (X - M \cdot x) \) with the full initial phase space distribution instead of Eq. (3.7):

\[ IR_{x} = \frac{\sqrt{\langle (X - M \cdot x)^2 \rangle}}{M}, \quad IR_{y} = \frac{\sqrt{\langle (Y - M \cdot y)^2 \rangle}}{M}. \]  

(3.15)

Qualitatively it is similar to the off-axis RMS resolution (dispersion of the LSF) but averaged over the whole field of view (or — whole initial phase space distribution, \( f(x) \)): \( IR \sim \int \int R_{x,y} f(x) dx \). Although the \( IR \) integral resolution is not as well defined as the \( AR \) and \( OR \) resolutions and gives rather conservative values, it may be useful as a single rough performance characteristic for an imaging system.

### 3.2.2 Chromatic Resolution

The proton beam is matched to a Russian quadruplet when the slopes of the beam emittance ellipses at the object plane, \( W_{x}, W_{y} \) are equal to the following ratios of the second order transfer map elements [24]:

\[ W_{x} = \frac{\langle xa \rangle}{\langle x^2 \rangle} = -\frac{M_{1,[1,6]}}{M_{1,[2,6]}}, \quad W_{y} = \frac{\langle yb \rangle}{\langle y^2 \rangle} = -\frac{M_{3,[3,6]}}{M_{3,[4,6]}}. \]  

(3.16)

In this case a Fourier plane is formed in the middle of the imaging lens where protons are radially sorted by their MCS angles, and the most significant second-order position dependent chromatic aberration caused by the \( M_{1,[1,6]} \) and \( M_{3,[3,6]} \) terms vanishes (strictly speaking, this aberration is fully canceled only for a matched beam with zero emittance). The remaining second order chromatic aberration depends only on the angles \( (a, b) \) and the energy \( (k) \) of a proton. The RMS resolution of the system at the object plane due to this aberration can then be approximated by the following expression:

\[ CR_{x,y} \approx 0.479 \cdot C_{x,y} \cdot c_{k}, \quad C_{x} = \frac{M_{1,[2,6]}}{M}, \quad C_{y} = \frac{M_{3,[4,6]}}{M}. \]  

(3.17)

The quantities \( C_{x,y} \) are called chromatic lengths of the system and reflect its intrinsic property. The coefficient 0.479 is due to the assumption that the collimator acceptance angle is matched to the object thickness so that \( \theta_c \approx \sigma_{a,b} \). We will call the approximation Eq. (3.17) chromatic resolution. If the beam after an object has a significant energy spread \( \sigma_{k} = \sqrt{\langle k^2 \rangle} \) and the collimator acceptance angle \( \theta_c \) is not too large, the chromatic resolution shall be the dominating term.

It is interesting to compare the values of \( AR \), \( OR \) and \( CR \) resolutions for the same system for different \( \theta_{c} \) and \( \sigma_{k} \). In Fig. 3.3 all these resolutions for one of the PRIOR-II designs are shown as a function of the beam energy spread. Obviously even with zero energy spread the \( AR \) and \( OR \) resolutions have finite values which are growing with the collimator acceptance angle. This is due the non-negligible pure geometrical aberrations of the magnifier, mostly — the third-order spherical aberration \( \frac{\partial^2 X}{\partial a^2} \). As it
3.2 Determination of Spatial Resolution

\[
\sigma_k = 0, \quad \sigma_k = 5 \cdot 10^{-4}, \quad \sigma_k = 2 \cdot 10^{-3}, \quad \sigma_k = 1 \cdot 10^{-3}
\]

\[
\begin{align*}
\theta_c &= 1 \text{ mrad} \quad \theta_c &= 3 \text{ mrad} \quad \theta_c &= 5 \text{ mrad} \\
\theta_c &= 7 \text{ mrad}
\end{align*}
\]

Figure 3.3: RMS resolutions of the PRIOR microscope as a function of the beam energy spread \(\sigma_k\) for different collimator angles \(\theta_c\). Solid lines — on-axis LSF dispersion (AR), dashed lines — off-axis LSF dispersion \((OR, x_0, y_0 = 10 \text{ mm}, 10 \text{ mm})\), dash-dotted lines — chromatic resolution \((CR)\).

\[
\begin{align*}
\sigma_k &= 0 \quad \sigma_k &= 5 \cdot 10^{-4} \quad \sigma_k &= 2 \cdot 10^{-3} \quad \sigma_k &= 1 \cdot 10^{-3}
\end{align*}
\]

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Figure 3.4: RMS resolutions of the PRIOR microscope as a function of the collimator angle \(\theta_c\) for different beam energy spreads \(\sigma_k\). See caption of Fig. 3.3.

should be, the \(OR\) values are always larger than \(AR\) due to the remaining position-dependent high-order aberrations. For large values of the beam energy spread \(\sigma_k > 2 \cdot 10^{-3}\) the chromatic resolution, \(CR\) (dash-dotted lines) describes the magnifier pretty well up to \(\theta_c = 5 \text{ mrad}\). However when the energy spread is small \((\sigma_k < 1 \cdot 10^{-3})\), the influence of the third-order geometrical and chromatic aberrations is clearly visible for \(\theta_c > 2 \text{ mrad}\).

The same data is shown in Fig. 3.4 but as a function of the collimator angle. Again, when the energy spread is large \((\sigma_k = 2 \cdot 10^{-3})\) the linear approximation of the chromatic resolution can well be employed to characterize the microscope even if a large-acceptance collimator is used. For smaller energy spreads
(which are more typical for the 4-5 GeV SIS-18 / SIS-100 proton beams and / or thin targets) or for large scattering angles (thick targets) the high order geometrical and mixed aberrations have to be taken into account which lead to a non-linear degradation of the spatial resolution. It can also be concluded that due to the intrinsic geometrical aberrations, one cannot infinitely enhance the resolution of a proton imaging system by increasing the proton energy $E_o$ (consequently reducing the relative energy spread after the object) if a finite-acceptance collimator is used which is matched to the amount of scattering in the object.

The developed method for the analytical calculation of the RMS resolution as the dispersion of a LSF is definitely a very powerful tool for designing and optimizing a proton microscope. However the RMS resolution alone does not fully characterize the imaging properties of the system: at the presence of the third and higher order aberrations the LSF often has a close to exponential shape rather than normal (see Fig. 3.2, left). The pure Gaussian image blur accrues for a focused image only when the aberrations are given mostly by the second-order chromatic terms $M_{1,2,6}$ or $M_{1,1,6}$. Since we are able to calculate any power moment of a LSF analytically, this can also be checked by calculating the LSF’s ”halo” parameter,

$$h = \frac{\langle X^4 \rangle}{\langle X^2 \rangle^2}. \quad (3.18)$$

For the normal distribution $h = 3$, for the truncated normal (Eq. (3.9)) $h \approx 2.09$ and for an equivalent parabolic $h \approx 2.14$ but for the PRIOR microscope LSFs when the beam energy spread is not too large and higher order aberrations are taken into account $h = 5 - 7$. On the one hand, a strongly peaked exponential LSF would result in a sharper image than could be expected just from the LSF’s dispersion (the RMS resolution). On the other hand, the longer tails of the LSF (the ”halo”) may affect the accuracy of the density reconstruction.

### 3.3 Ion Optical Design

A schematic layout of the PRIOR microscope is shown in Fig. 3.5. For the given total length of the system $L_{tot}$, proton energy $E_o$ and maximum field gradient of the quadrupoles, the distances $L_{1-4}$ and $Q_{1-2}$ ($\sum L = L_{tot}$) have to be optimized to achieve the best performance of the microscope. For this purpose a second-order thin lens approximation model has been developed [16]. This analytical model allows to quickly study and optimize the performance of the microscope as a function of its geometry.

![Figure 3.5: Schematic layout of a proton microscope.](image)

After the principle behavior of the system such as dependencies of chromatic lengths and matching parameters (see sec. 3.2.2, p. 21) on the geometry have been studied analytically [16], the performance of the microscope has been simulated and optimized using the arbitrary-order ion optical code COSY INFINITY v.9.1 [39, 40]. The simulations have been performed using 3rd-order transfer maps and realistic fringe field models for all the elements. The goal of the optimization was to achieve the best spatial resolution which was calculated analytically with a COSY script as described in sec. 3.2 above, while fulfilling the matching conditions and obtaining a proper FOV. The PRIOR-II design has been optimized for both the experiments at FAIR (APPA cave) and at GSI (HHT cave).
3.3 Ion Optical Design

3.3.1 PRIOR-II at FAIR

The original design of the HEDgeHOB SIS-100 beam line at FAIR has been made for the HIHEX and LAPLAS experiments with intense uranium beams and did not foresee a proton microscope installation. A strong superconducting final focusing system (FFS) is installed at the end of the line for these experiments. For the PRIOR experiments, the whole matching section of the line upstream the FFS quadrupoles had to be completely redesigned so that it will be able to match uranium beams for HIHEX and LAPLAS as well as proton beams for PRIOR-II.

The new layout of the HEDgeHOB beam line is shown in Fig. 3.6. The PRIOR-II magnets and the experiment are installed in the 6.9 m-long gap upstream the FFS. This gap is introduced for both installing the LAPLAS wobbler and matching the uranium beams to the FFS [42]. For the PRIOR experiments, the FFS is switched off and is used just as a drift space. The total length of the microscope is \( L_{\text{tot}} = 20 \text{ m} \). The new matching section consisting of two quadruplets is pretty flexible and it is able to match the beams for all of the experiments. In the PRIOR "mode", the second quadruplet QD31-QD42 is matching the proton beam to the microscope while the first quadruplet is used to expand the beam (see Fig. 3.7).

The fielding of the PRIOR-II setup in the beam line is shown in Fig. 3.8. The PRIOR magnets are installed in the "wobbler gap" (between two RF waveguides painted in light green) and the orange magnets upstream belong to the second matching quadruplet. Since the length of the PRIOR-II setup from the object till the end of the fourth quadrupole is 3.3 m, the 6.9 m "wobbler gap" provides enough space for installing an experimental setup to be used with PRIOR.

3.3.2 PRIOR-II at GSI

The layout of the HHT beam line at GSI with the PRIOR-II microscope fielded in the HHT cave is shown in Fig. 3.9. The matching in this case is provided by the last five quadrupoles of the HHT line in the
3.3 Ion Optical Design

Figure 3.8: Fielding of the PRIOR-II setup in the APPA cave at FAIR. The PRIOR experiment and detector are now shown.

The same way as it has been done for the PRIOR-I prototype (see Fig. 2.1, p. 4). Although the length of the PRIOR-II magnifier is slightly larger than the length of the prototype, the available distance between the last matching quadrupole and the cave’s wall (6.2 m) is sufficient for installing both the magnifier and an experiment is the cave.

Figure 3.9: Layout of the HHT beam line at GSI with the PRIOR-II microscope. Only the last five quadrupoles of the matching section are shown.

The ion-optical layout of the whole HHT beam line is shown in Fig 3.10 along with the envelopes of the matched beam. Here the apertures of the last two matching quadrupoles are limiting the FOV of the system. The maximum achievable magnification of the system is limited by the total length available in the HHT area, $L_{\text{tot}} = 9.5 \text{ m}$.

Figure 3.10: Ion optical layout of the HHT beam line at GSI (see also Fig. 3.9) with the PRIOR-II microscope. The beam envelopes show the matched beam.
Finally, the 3D CAD model of the PRIOR-II microscope fielded at the HHT area of GSI is shown in Fig. 3.11. On the top image one can also see the UEWE experimental installation (sec. 2.3) together with the PRIOR-II setup.

Figure 3.11: Fielding of the PRIOR-II setup in the HHT cave at GSI.
3.4 Influence of Field Quality and Stability

Due to the symmetry of a quadrupole electromagnet (in contrast to e.g., a Halbach-type PMQ) the first remaining parasitic field harmonics are $B_6$, $B_{10}$ and $B_{14}$. All the lower parasitic harmonics are vanishing by design and may only appear at the $10^{-6} - 10^{-5}$ level due to manufacturing and alignment errors. In order to determine which field quality is needed for the PRIOR-II quadrupoles, the influence of the field harmonics on the RMS resolution has been evaluated. The strongest of the remaining harmonics is $B_6$. Therefore the degradation of the RMS resolution as a function of the integral $b_6 = B_6 / B_2$ harmonic magnitude has been calculated for a reference ion-optical design of the microscope. As the $n^{th}$ field harmonic affects transfer map elements starting from the order $n - 1$, the calculations have been performed using the $5^{th}$-order transfer map. The results of these calculations are shown in Fig. 3.12.

As the $b_6$ field harmonic mostly affects the $5^{th}$-order position-dependent elements of the map, the $AR$ resolution considerably underestimates the effect. The $IR$ resolution would overestimate the influence of the field quality since it averages the effect over the whole field of view, and the $OR$ resolution should give the correct result for a single off-axis point. Hence the true scaling is between the $OR$ and $IR$ curves. One can see that in order to keep the resolution degradation at the few percent level, the field quality shall be about $3 - 5 \times 10^{-4}$ or below.

The stability of the field in the lenses is mainly given by the stability of the driving current, i.e. — the stabilization of the power converters. When the fields in the lenses of a perfectly focused microscope spontaneously change, the image planes move off the detector plane and also the violated matching conditions (Eq. (3.16)) lead to a degradation of the resolution due to the not canceled second-order chromatic terms. These are strong and linear effects in a sense that the shape of the resulting LSF will be fully determined by the initial distributions $f_{ab}(a, b)$ and $f_k(k)$. The latter can be checked by calculating the LSF halo parameter as a function of the relative field changes (see Fig. 3.13). When the microscope is perfectly focused, the LSF has long tails due to high-order aberrations and the halo parameter $h \approx 6.5$. With the change of a lens’ current the second-order chromatic aberrations become stronger and the microscope moves out of focus. The halo parameter rapidly decreases passing the $h = 3$ value of the normal distribution and finally becomes $h \approx 2.14$, characterizing the initial angular distribution which is this particular case was assumed to be parabolic.

The influence of the magnet current stabilization level on the RMS resolution is shown in Fig. 3.14. One can see that for large current changes all the relative resolutions have the same scaling indicating that

![Figure 3.12: RMS resolutions degradation as a function of the $b_6$ field harmonic. $AR$ — on-axis LSF dispersion, $OR$ — off-axis LSF dispersion at (10 mm, 10 mm), $IR$ — integral resolution (Eq. (3.15)).](image-url)
3.4 Influence of Field Quality and Stability

![Graph showing the relationship between current stabilization level and LSF halo factor.](image1)

Figure 3.13: LSF halo parameter (Eq. (3.18)) as a function of relative field fluctuation for a perfectly focused proton microscope.

![Graph showing RMS resolutions degradation as a function of the current fluctuations level.](image2)

Figure 3.14: RMS resolutions degradation as a function of the current fluctuations level. $AR$ — on-axis LSF dispersion, $OR$ — off-axis LSF dispersion at (10 mm, 10 mm), $IR$ — integral resolution.

The first-order "defocussing" effect dominates. At smaller changes ($< 10^{-4}$) the second and higher order position-dependent aberrations are still important and therefore $AR$ underestimates the RMS resolution degradation. From the Fig. 3.14 it can be concluded that in order to keep the resolution degradation at a sub-percent level (which is more than sufficient) one needs stabilize the currents down to the $1 - 2 \cdot 10^{-5}$ level. A better stabilization is probably not required because the effects of the room temperature fluctuations are expected to be at the $10^{-5}$ level as well."
3.5 Summary and Discussion

As the result of the design work described in this chapter, the key parameters of the PRIOR-II microscope are summarized in Table 3.2. The table shows parameters of the same system fielded in the HHT cave of GSI as well as in the APPA cave of FAIR. The main difference between the two cases is the available object-to-image distance, \( L_{\text{tot}} \). In the HHT cave it is limited to 9.5 m whereas in the APPA cave it can be set to 20 m or more.

Table 3.2: Ion optical design and performance of PRIOR-II at FAIR and at GSI. The resolutions have been calculated under assumption that the after the object the energy and angular spreads of the beam are \( \sigma_k = 7.1 \cdot 10^{-4} \) and \( \sigma_{a,b} = \theta_c \), respectively.

<table>
<thead>
<tr>
<th></th>
<th>FAIR</th>
<th>GSI</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Geometry</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Object to image distance, ( L_{\text{tot}} ) (m)</td>
<td>20.0</td>
<td>9.5</td>
</tr>
<tr>
<td>Object to matching distance, ( L_0 ) (m)</td>
<td>1.0</td>
<td>1.0</td>
</tr>
<tr>
<td>Stand-off, ( L_1 ) (m)</td>
<td>0.5</td>
<td>0.5</td>
</tr>
<tr>
<td>Outer gaps Q1-Q2 and Q3-Q4, ( L_2 ) (m)</td>
<td>0.1</td>
<td>0.1</td>
</tr>
<tr>
<td>Middle gap Q2-Q3, ( L_3 ) (m)</td>
<td>0.5</td>
<td>0.4</td>
</tr>
<tr>
<td>Distance to the image, ( L_4 ) (m)</td>
<td>16.7</td>
<td>6.3</td>
</tr>
<tr>
<td><strong>Optical properties</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Reference energy, ( E_0 ) (MeV)</td>
<td>5000</td>
<td>4000</td>
</tr>
<tr>
<td>Normalized RMS emittance, ( \epsilon_x \times \epsilon_y ) (mm-mrad)</td>
<td>6.25 x 2</td>
<td>6.25 x 2.5</td>
</tr>
<tr>
<td>Magnification, ( M )</td>
<td>8.03</td>
<td>3.49</td>
</tr>
<tr>
<td>Chromatic length, ( C_x \times C_y ) (m)</td>
<td>2.66 x 5.04</td>
<td>2.77 x 5.24</td>
</tr>
<tr>
<td>Matched beam slope, ( W_x \times W_y ) (mrad/mm)</td>
<td>-0.4849 x -0.2270</td>
<td>-0.5292 x -0.1990</td>
</tr>
<tr>
<td>Location of Fourier plane, ( L_{F_x} \times L_{F_y} ) (mm)</td>
<td>-80 x -50</td>
<td>-80 x -47</td>
</tr>
<tr>
<td>Angular dispersion, ( D_x \times D_y ) (mm/mrad)</td>
<td>1.36 x 2.52</td>
<td>1.31 x 2.34</td>
</tr>
<tr>
<td><strong>Performance</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Collimator acceptance, ( \theta_c ) (mrad)</td>
<td>2</td>
<td>5</td>
</tr>
<tr>
<td>Field of view, ( FOV_x \times FOV_y ) (mm)</td>
<td>30 x 52</td>
<td>29 x 48</td>
</tr>
<tr>
<td>Chromatic RMS resolution, ( CR_x \times CR_y ) (µm)</td>
<td>1.8 x 3.4</td>
<td>4.5 x 8.5</td>
</tr>
<tr>
<td>Off-axis RMS resolution, ( OR_x \times OR_y ) (µm)</td>
<td>2.0 x 3.9</td>
<td>5.1 x 9.7</td>
</tr>
</tbody>
</table>

Despite a big difference in magnification, both systems demonstrate quite similar performance in terms of the field of view and the resolution. The resolution values shown in Table 3.2 indicate the best achievable resolutions for the given \( \theta_c \) and \( \sigma_k \), limited by the ion-optical aberrations of the magnifier only. It is to be noted that in the both cases, at GSI and at FAIR the real resolutions are likely to be limited by the detector resolution and not by the aberrations. The resolution of a detector depends on the pixel resolution of the camera used, on the optical resolution of the lenses / objectives imaging the scintillator to the camera’s chip and on the PSF width of the scintillator itself. The latter two limits are hard to make smaller than 30-50 µm. Therefore at GSI with the relatively small magnification of \( M = 3.5 \) the detector-limited resolution at the object plane will be \( \approx 9 - 15 \) µm. A 6-10 Megapixels (Mp) camera is required.

At FAIR with the magnification of \( M = 8 \) the scintillator PSF and light imaging optics limit the resolution at the object plane only at 3-6 µm level which is comparable to the aberrations limit (Tab. 3.2) or even smaller. However with such a resolution it is difficult to cover the whole FOV because of a limited pixel resolution: a 25-50 Mp chip will be required. Assuming a 12 Mp camera, the resolution limit for the whole FOV will be about 9-12 µm while a 5 µm limit can be reached only if half of the available FOV is
The PRIOR-II design for the both cases assumes a certain degree of flexibility: the performance and properties of the magnifier can be adapted to an experiment by changing the $L_1$ and $L_4$ distances, especially — at FAIR. The FOV can also be enlarged by increasing the $L_3$ distance at the cost of slightly degraded spatial resolution of the system.
4 Design and Technical Specifications of the PRIOR Quadrupole Magnets

4.1 Requirements

Based on the results of Chapter 3, a detailed design of the PRIOR-II electromagnets has been carried out using the OPERA code [43]. The main requirements for the quadrupoles are summarized in Table 4.1. As follows from the ion optical design and optimization, the required pole tip radius of the magnets is $R_p = 30$ mm, and the required lengths were slightly increased to $L_{Q1} = 40$ cm for the "short" (Q1), outer quadrupoles and $L_{Q2} = 65$ cm for the "long" (Q2), inner ones. The pole tip field should be as high as possible, retaining the required quality of the field. The field quality should be optimized for the field gradients between 80% and 90% load ("working" field gradient), which roughly corresponds to the 4-5 GeV proton energies.

Table 4.1: Main requirements for the PRIOR-II quadrupole magnets.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pole tip radius, $R_p$</td>
<td>30 mm</td>
</tr>
<tr>
<td>Yoke length:</td>
<td></td>
</tr>
<tr>
<td>&quot;short&quot;, $L_{Q1}$</td>
<td>40 cm</td>
</tr>
<tr>
<td>&quot;long&quot;, $L_{Q2}$</td>
<td>65 cm</td>
</tr>
<tr>
<td>Maximum pole tip field, $B_{\text{max}}$</td>
<td>1.3 T</td>
</tr>
<tr>
<td>Maximum field gradient, $G_{\text{max}}$</td>
<td>43.3 T/m</td>
</tr>
<tr>
<td>Working field gradient, $G_w$</td>
<td>35 – 39 T/m</td>
</tr>
<tr>
<td>Good field radius, $R_{\text{GF}}$</td>
<td>25 mm</td>
</tr>
<tr>
<td>Field quality</td>
<td>$\pm (3 - 5) \cdot 10^{-4}$</td>
</tr>
</tbody>
</table>

4.2 2D Model of the Quadrupole

The cross section of the 2D quadrupole model is shown in Fig. 4.1. The inner radius of the yoke is 200 mm, the outer radius is 280 mm. The properties of the yoke material correspond to the ARMCO steel as they are defined in the OPERA software.

In order to improve the field quality at high flux density levels, a round hole in the yoke (Purcell filter [44]) was introduced (see Fig. 4.1). The hole has a diameter of 8 mm and the distance between the center of the hole and the pole tip is 12.5 mm.

The digitized shape of the pole tip is presented in Table 4.2. The shape of the yoke has been optimized to achieve the best field quality, especially in the region of the working field gradients, $G_w$. For this purpose the simulations were performed for three different magnet loads: "low" (14%), "working" (85%) and "high" (102%), see Table 4.3. The corresponding distributions of the flux density at the magnet central plane are shown in Fig. 4.2. One can see that the maximum relative field deviations for working and high loads are smaller than $\pm 3 \cdot 10^{-4}$. 

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4.2 2D Model of the Quadrupole

Figure 4.1: Cross section of the quadrupole (left) and its yoke (right).

Table 4.2: The pole profile.

<table>
<thead>
<tr>
<th>x, mm</th>
<th>y, mm</th>
<th>x, mm</th>
<th>y, mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>42.9885</td>
<td>10.4521</td>
<td>31.0714</td>
<td>14.4778</td>
</tr>
<tr>
<td>42.2911</td>
<td>10.3309</td>
<td>30.4364</td>
<td>14.7840</td>
</tr>
<tr>
<td>41.6023</td>
<td>10.4063</td>
<td>29.8061</td>
<td>15.0997</td>
</tr>
<tr>
<td>40.9326</td>
<td>10.6312</td>
<td>29.1808</td>
<td>15.4254</td>
</tr>
<tr>
<td>40.2675</td>
<td>10.8651</td>
<td>28.5610</td>
<td>15.7614</td>
</tr>
<tr>
<td>39.6026</td>
<td>11.0993</td>
<td>27.9472</td>
<td>16.1081</td>
</tr>
<tr>
<td>38.9239</td>
<td>11.3396</td>
<td>27.3398</td>
<td>16.4660</td>
</tr>
<tr>
<td>38.2599</td>
<td>11.5766</td>
<td>26.7394</td>
<td>16.8355</td>
</tr>
<tr>
<td>37.5967</td>
<td>11.8159</td>
<td>26.1338</td>
<td>17.2249</td>
</tr>
<tr>
<td>36.9347</td>
<td>12.0582</td>
<td>25.5489</td>
<td>17.6184</td>
</tr>
<tr>
<td>36.2740</td>
<td>12.3041</td>
<td>24.9724</td>
<td>18.0242</td>
</tr>
<tr>
<td>35.6149</td>
<td>12.5543</td>
<td>24.4048</td>
<td>18.4424</td>
</tr>
<tr>
<td>34.9577</td>
<td>12.8095</td>
<td>23.8467</td>
<td>18.8731</td>
</tr>
<tr>
<td>34.3027</td>
<td>13.0702</td>
<td>23.2985</td>
<td>19.3163</td>
</tr>
<tr>
<td>33.6501</td>
<td>13.3371</td>
<td>22.7606</td>
<td>19.7720</td>
</tr>
<tr>
<td>33.0004</td>
<td>13.6107</td>
<td>22.2335</td>
<td>20.2402</td>
</tr>
<tr>
<td>32.3538</td>
<td>13.8917</td>
<td>21.7176</td>
<td>20.7207</td>
</tr>
</tbody>
</table>

Table 4.3: Load levels of the magnet used during the optimization.

<table>
<thead>
<tr>
<th>Load</th>
<th>Level</th>
<th>Field gradient</th>
<th>Pole tip field</th>
</tr>
</thead>
<tbody>
<tr>
<td>low, $G_l$</td>
<td>14%</td>
<td>6 T/m</td>
<td>0.18 T</td>
</tr>
<tr>
<td>working, $G_w$</td>
<td>85%</td>
<td>37 T/m</td>
<td>1.11 T</td>
</tr>
<tr>
<td>high, $G_h$</td>
<td>102%</td>
<td>44 T/m</td>
<td>1.32 T</td>
</tr>
</tbody>
</table>
4.3 3D Model and Integral Properties

Figure 4.2: The flux density distribution in the magnet central plane at \( r = R_{GF} \) for different load levels (see Table 4.3). Left: low load, \( \frac{\Delta G}{G} \) \( \leq 4 \cdot 10^{-4} \). Middle: working load, \( \frac{\Delta G}{G} \) \( \leq 2.7 \cdot 10^{-4} \). Right: high load \( \frac{\Delta G}{G} \) \( \leq 2.4 \cdot 10^{-4} \).

4.3 3D Model and Integral Properties

Figure 4.3: 3D model of the quadrupole. Left: front view, middle: side view, right: the yoke.

Figure 4.4: The parameters of the end chamfer.

A 3D view of the magnets is shown in Fig. 4.3 and geometry of the end chamfer — in Fig. 4.4. The integral properties of the quadrupole are similar to those obtained in the 2D optimization (or calculated in the central cross section). The integral field gradient \( G \) and integral field harmonic \( B_n \) are calculated
as

\[ G = \frac{\int g(z) \, dz}{L} \quad \text{and} \quad B_n = \frac{\int b_n(z) \, dz}{L}, \]

where \( L \) is the length of the magnet.

Figure 4.5: Integral flux density distribution at \( r = R_{GF} \) for "short" (Q1, left) and "long" (Q2, right) quadrupoles.

The integral flux density distribution for different load levels is shown in Fig. 4.5. Again one can see that the maximum relative field deviation for the working load \((G = 37 \, \text{T/m})\) is better than \(\pm 3 \cdot 10^{-4}\), and for low and high loads — better than \(\pm 4 - 5 \cdot 10^{-4}\).

Figure 4.6: Integral field harmonics at \( r = R_{GF} \) Q1 (left) and Q2 (right) quadrupoles as a function of the pole tip field.

Due to the symmetry of the magnet, the only remaining parasitic field harmonics \( B_n \) are with \( n = 6, 10 \) and 14. The magnet design has been optimized in terms of the field quality so that at the working load \((B_w \approx 1.1 \, \text{T})\) the relative amplitude of the integral 6th harmonic \(B_6/B_2 < 1 \cdot 10^{-4}\) (see Fig. 4.6).

The longitudinal distributions of the field gradient (fringe fields) at different loads for the Q1 and Q2 magnets are shown in Figs. 4.7 and 4.8 respectively. Finally, the analogous longitudinal distributions of the parasitic field harmonics are shown in Figs. 4.9 and 4.10.

### 4.4 Coils and Power Converters

The coil of the magnet will be fabricated using the radiation-resistant copper cable which was specially designed for the SIS-100 quadrupoles. The cross section of the coil consisting of 14 turns is shown in
4.4 Coils and Power Converters

Figure 4.7: Longitudinal distribution of the field gradient at \( r = R_{GF} \) for the Q1 ("short") quadrupole. Left: low load, middle: working load, right: high load.

Figure 4.8: Longitudinal distribution of the field gradient at \( r = R_{GF} \) for the Q2 ("long") quadrupole. Left: low load, middle: working load, right: high load.

Figure 4.9: Longitudinal distribution of the parasitic field harmonics for the Q1 ("short") quadrupole. Left: low load, middle: working load, right: high load.

Figure 4.10: Longitudinal distribution of the parasitic field harmonics for the Q2 ("long") quadrupole. Left: low load, middle: working load, right: high load.

Fig. 4.11: The main parameters of the coil and the cable common for the Q1 and Q2 magnets are given in Table 4.4.

In order to achieve the maximum pole tip field of 1.3 T (integral field gradient of 43.3 T/m) a current of 1.7 kA is needed from the power converters at the voltages on the magnets of 17 V and 24 V for the Q1 and Q2 lenses, respectively. The load lines (excitation curves) for both quadrupoles are shown in
4.5 Technical Specifications Summary

The technical specifications for the PRIOR-II quadrupole magnets Q1 ("short") and Q2 ("long") are summarized in Table 4.5. For the common parameters of the coil see also Table 4.4.
### 4.5 Technical Specifications Summary

Table 4.5: Parameters of the PRIOR-II quadrupoles.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Q1</th>
<th>Q2</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>General design</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pole tip radius</td>
<td>30 mm</td>
<td></td>
</tr>
<tr>
<td>Good field radius</td>
<td>25 mm</td>
<td></td>
</tr>
<tr>
<td>Maximum pole tip field</td>
<td>1.3 T</td>
<td></td>
</tr>
<tr>
<td>Maximum field gradient</td>
<td>43.3 T/m</td>
<td></td>
</tr>
<tr>
<td>Field quality at 85% load</td>
<td>$&lt; \pm 3 \cdot 10^{-4}$</td>
<td></td>
</tr>
<tr>
<td>Total length (yoke and coil)</td>
<td>48 cm</td>
<td>73 cm</td>
</tr>
<tr>
<td>Total weight</td>
<td>711 kg</td>
<td>1137 kg</td>
</tr>
<tr>
<td><strong>Yoke</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Material</td>
<td>ARMCO steel</td>
<td></td>
</tr>
<tr>
<td>Inner radius</td>
<td>200 mm</td>
<td></td>
</tr>
<tr>
<td>Outer radius</td>
<td>280 mm</td>
<td></td>
</tr>
<tr>
<td>Length</td>
<td>40 cm</td>
<td>65 cm</td>
</tr>
<tr>
<td>Weight</td>
<td>626 kg</td>
<td>1020 kg</td>
</tr>
<tr>
<td><strong>Coil</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cable cross section</td>
<td>$13 \times 13 , \text{mm}^2$</td>
<td></td>
</tr>
<tr>
<td>Turns per coil</td>
<td>14</td>
<td></td>
</tr>
<tr>
<td>Average current density</td>
<td>8.05 A/mm$^2$</td>
<td></td>
</tr>
<tr>
<td>Current density in the conductor</td>
<td>13.0 A/mm$^2$</td>
<td></td>
</tr>
<tr>
<td>Water pressure</td>
<td>2.6 bar</td>
<td>6.2 bar</td>
</tr>
<tr>
<td>Water consumption</td>
<td>0.35 l/s</td>
<td>0.41 l/s</td>
</tr>
<tr>
<td>Temperature drop</td>
<td>20°C</td>
<td></td>
</tr>
<tr>
<td>Copper weight</td>
<td>85 kg</td>
<td>117 kg</td>
</tr>
<tr>
<td><strong>Power converter</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Current</td>
<td>1.7 kA</td>
<td></td>
</tr>
<tr>
<td>Current stabilization level</td>
<td>$1 - 2 \cdot 10^{-5}$</td>
<td></td>
</tr>
<tr>
<td>Voltage on magnet</td>
<td>17 V</td>
<td>24 V</td>
</tr>
<tr>
<td>Power</td>
<td>29 kW</td>
<td>41 kW</td>
</tr>
<tr>
<td>Required input power</td>
<td>40 kVA</td>
<td>55 kVA</td>
</tr>
</tbody>
</table>
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