REFLECTION MATRIX DEFINITIONS

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Abstract

This document defines a notation for some elementary reflection matrices.

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Mots-clés: matrice, matrice de réflexion, réflexion

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TABLE OF CONTENTS

1. INTRODUCTION ................................................................................................................. 1

2. ELEMENTARY 2D REFLECTION MATRICES ...................................................................... 1
   2.1 2D Matrix for the Reflection in the Line x = 0 ................................................................. 1
   2.2 2D Matrix for the Reflection in the Line y = 0 ................................................................. 1
   2.3 2D Matrix for the Reflection in the Line x = y ................................................................. 1

3. ELEMENTARY 3D REFLECTION MATRICES ...................................................................... 1
   3.1 3D Matrix for the Reflection in the Plane x = 0 ............................................................... 2
   3.2 3D Matrix for the Reflection in the Plane y = 0 ............................................................... 2
   3.3 3D Matrix for the Reflection in the Plane z = 0 ............................................................... 2
   3.4 3D Matrix for the Reflection in the Plane x = y ............................................................... 2
   3.5 3D Matrix for the Reflection in the Plane x = z ............................................................... 2
   3.6 3D Matrix for the Reflection in the Plane y = z ............................................................... 3
1. INTRODUCTION

This document defines a notation for some elementary reflection matrices. These matrices may be applied when transforming vectors and coordinates between a left-handed and a right-handed Cartesian coordinate system.

The matrices defined here are all improper orthogonal matrices. They are also all symmetrical so they are their own inverse.

The same notation will be used for the 2 dimensional reflection matrices, and those defined in 3 dimensions that are generalisations of the same cases.

2. ELEMENTARY 2D REFLECTION MATRICES

There are three elementary reflection matrices in two dimensions. They are each defined below.

2.1 2D Matrix for the Reflection in the Line $x = 0$

The 2D matrix for the reflection in the line, $x = 0$, is defined as follows,

$$
P_x = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \quad (1)
$$

2.2 2D Matrix for the Reflection in the Line $y = 0$

The 2D matrix for the reflection in the line, $y = 0$, is defined as follows,

$$
P_y = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \quad (2)
$$

2.3 2D Matrix for the Reflection in the Line $x = y$

The 2D matrix for the reflection in the line, $x = y$, is defined as follows,

$$
P_{xy} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \quad (3)
$$

3. ELEMENTARY 3D REFLECTION MATRICES

There are six elementary reflection matrices in three dimensions, three of which are generalisations of the 2 dimensional matrices.
3.1 3D Matrix for the Reflection in the Plane \( x = 0 \)

The 3D matrix for the reflection in the plane, \( x = 0 \), is defined as follows,

\[
P_x = \begin{bmatrix}
-1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1 \\
\end{bmatrix}
\]  

(4)

3.2 3D Matrix for the Reflection in the Plane \( y = 0 \)

The 3D matrix for the reflection in the plane, \( y = 0 \), is defined as follows,

\[
P_y = \begin{bmatrix}
1 & 0 & 0 \\
0 & -1 & 0 \\
0 & 0 & 1 \\
\end{bmatrix}
\]  

(5)

3.3 3D Matrix for the Reflection in the Plane \( z = 0 \)

The 3D matrix for the reflection in the plane, \( z = 0 \), is defined as follows,

\[
P_z = \begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & -1 \\
\end{bmatrix}
\]  

(6)

3.4 3D Matrix for the Reflection in the Plane \( x = y \)

The 3D matrix for the reflection in the plane, \( x = y \), is defined as follows,

\[
P_{x y} = \begin{bmatrix}
0 & 1 & 0 \\
1 & 0 & 0 \\
0 & 0 & 1 \\
\end{bmatrix}
\]  

(7)

3.5 3D Matrix for the Reflection in the Plane \( x = z \)

The 3D matrix for the reflection in the plane, \( x = z \), is defined as follows,

\[
P_{x z} = \begin{bmatrix}
0 & 0 & 1 \\
0 & 1 & 0 \\
1 & 0 & 0 \\
\end{bmatrix}
\]  

(8)
3.6 3D Matrix for the Reflection in the Plane $y = z$

The 3D matrix for the reflection in the plane, $y = z$, is defined as follows,

$$P_{yz} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \quad (9)$$